Harmonic Analysis

Harmonic Analysis

VIC circuits are typically driven by non-sinusoidal waveforms (pulses, square waves), which contain harmonics. Understanding how these harmonics interact with the resonant circuit is important for predicting actual performance and potential interference effects.

Fourier Analysis Basics

Any periodic waveform can be decomposed into a sum of sinusoids:

Fourier Series:

$f(t) = a? + ?[a?\cos(n?t) + b?\sin(n?t)]$

Where n = 1, 2, 3... are the harmonic numbers (n=1 is fundamental)

Harmonic Content of Common Waveforms

Square Wave

50% duty cycle square wave contains only odd harmonics:

$V(t) = (4V_{pk}/?)[\sin(?t) + (1/3)\sin(3?t) + (1/5)\sin(5?t) + ...]$

Harmonic	Frequency	Relative Amplitude
1st (fundamental)	f	100%
3rd	3f	33.3%

Harmonic	Frequency	Relative Amplitude
5th	5f	20%
7th	7f	14.3%

Pulse Train (Variable Duty Cycle)

Pulse train with duty cycle D contains both odd and even harmonics:

$a_n = (2V_{pk}/n?) \times \sin(n?D)$

Effect of Duty Cycle:

- **D = 50%:** Only odd harmonics (even harmonics cancel)
- **D = 25%:** Strong 2nd harmonic, weak 4th
- **D = 33%:** No 3rd harmonic (3rd harmonic null)
- **Narrow pulse:** Wide harmonic spectrum, many significant harmonics

Resonant Circuit Response to Harmonics

A resonant circuit acts as a bandpass filter. It responds most strongly to frequencies near f_0:

```
Response
    │
    │          Fundamental
    │              ↓
    │             /\
    │            /  \              3rd harmonic
    │           /    \                 ↓
    │          /      \              (small response)
    │         /        \           ┌─┐
    │        /          \          │ │
    └──────────────────────────────────────→ f
             f₀          3f₀
```

Response at Harmonic Frequencies:

$H(nf) = 1 / ?[1 + Q^2(n - 1/n)^2]$

For high Q circuits, harmonics far from f_0 are strongly attenuated.

Example (Q=50, f?=10 kHz):

> - At 10 kHz (1st): Response = 100%
> - At 30 kHz (3rd): Response ≈ 0.6%
> - At 50 kHz (5th): Response ≈ 0.2%

Harmonic Resonance

If a harmonic happens to fall near f_0, it can cause problems or opportunities:

Scenario	Effect	Action
Drive at f_0	Fundamental resonates	Normal operation
Drive at $f_0/2$	2nd harmonic resonates	May be useful or problematic
Drive at $f_0/3$	3rd harmonic resonates	Subharmonic driving
Harmonic hits SRF	Choke self-resonates	Avoid—causes problems

Sub-Harmonic Driving

It's possible to drive the circuit at a sub-multiple of f_0 and let a harmonic excite resonance:

Example: 3rd Harmonic Drive

- Circuit resonance: f_0 = 30 kHz
- Drive frequency: f_{drive} = 10 kHz
- 3rd harmonic of drive (30 kHz) excites resonance

Advantages:

- Lower switching frequency (easier on semiconductors)
- Different pulse characteristics
- May interact differently with WFC

Disadvantages:

- Harmonic has lower amplitude than fundamental
- Reduced efficiency (energy in unused harmonics)
- More complex analysis

Pulse Shaping for Harmonic Control

Adjusting pulse shape can control harmonic content:

Technique	Effect
Slower edges (rise/fall time)	Reduces high-order harmonics
Duty cycle = 1/n	Eliminates nth harmonic
Trapezoidal waveform	Controlled harmonic rolloff
Sine wave drive	No harmonics (pure fundamental)

Harmonic Interaction with Multiple Resonances

In dual-resonant VIC (primary + secondary), harmonics may interact with both:

```
Response
    |
    |     Primary        Secondary
    |     resonance      resonance
    |         ↓              ↓
    |        /\            /\
    |       /  \          /  \
    |      /    \        /    \
    |     /      \      /      \
    |    /        \    /        \
    |   /          \  /          \
    └──────────────────────────────────→ f
          f₀,pri       f₀,sec
```

If $f_{0,sec} = 3 \times f_{0,pri}$, then:

- Fundamental drives primary resonance
- 3rd harmonic drives secondary resonance
- This is sometimes called "harmonic matching"

Practical Harmonic Considerations

EMI Concerns:

Harmonics can cause electromagnetic interference. Shield appropriately and consider filtering if needed.

Measurement:

Use an oscilloscope with FFT function or spectrum analyzer to view harmonic content of your signals.

Design Rule:

For clean resonance, ensure no significant harmonics fall within the passband $(f_0 \pm f_0/Q)$ of unintended resonances.

Harmonic Analysis in VIC Matrix Calculator

Calculator Feature: The simulation can show frequency response across a range that includes harmonics. When analyzing a design, check whether any harmonics of your drive frequency fall near the circuit's resonant points or SRF values.

Next: Transformer Coupling Effects →

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